Polarization Effects in Superfluid $^4$He

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Part 1

SECOND SOUND
Electrical response to the second sound wave

Rybalko, Rudavskii, Tikhii et al 2004-2010

\[ \frac{\Delta T}{\Delta U} = 2.3 \times 10^4 \text{ K/V} \]

\[ \Delta U \] electrod

\[ \Delta T \] bolometer

\[ T(x) \]

\[ v_n \]

\[ v_s \]

\[ \text{heater} \]
Dipole moment of He atom due to the force \( f \) acting on electron shell of given atom from the side of surrounding atoms

\[
\hat{H}_e = \hat{H}_0 + (eE - f)(\rho_1 + \rho_2)
\]

\[
\rho_1 = r_1 - r_n
\]

\[
\rho_2 = r_2 - r_n
\]

\( f \) causes:

(i) centre of gravity accelerated motion,
(ii) electron shell deformation

\[
\Psi'_0 \approx \Psi_0 - \frac{\langle \Psi_0 | (eE - f)(\rho_1 + \rho_2) | \Psi_1 \rangle}{E_1 - E_0}
\]

\[
E'_0 - E_0 \approx -\frac{r_{at}^2 (eE - f)^2}{E_1 - E_0}
\]

\[
d = -\alpha f, \quad \alpha \approx \frac{2er_{at}^2}{E_1 - E_0}
\]

\[
r_{at} \approx 2.7 \text{ Å} \quad (E_1 - E_0) \approx 20 \text{ eV} \quad \alpha \approx 2.2 \times 10^{-14} \text{ CGSE units}
\]
First sound waves

\[ a = \frac{1}{\rho} \frac{\partial j}{\partial t} = -\frac{1}{\rho} \nabla P \]

\[ d = -\alpha f \quad \text{f} \propto (\omega \tau) m a \]

\[ E = -4\pi n d \]

\[ \Delta U = 4\pi \alpha (\omega \tau) \Delta P \]

\[ \Delta U_{min} = 3 \times 10^{-9} \text{ Volt } = 4\pi \alpha (\omega \tau) \Delta P_{min} \]

\[ \Delta P_{min} \approx 4 \text{ Atm} \]
Second sound waves

The slow relative accelerated motion of normal and superfluid components is accompanied by the fast process of establishing of local equilibrium in densities of normal and superfluid components. The interatomic collisions do not play a role in weakening of atomic dipole moments which we assume proportional to the relative acceleration of normal and superfluid liquids:

\[ f \propto m \frac{\partial (v_s - v_n)}{\partial t} \]

To illustrate this statement let us look at the roton microscopic structure.
Roton wave packet. Local momentum distribution

Galli, Cecchetti, Reatto 1996
Atomic dipole moments inside roton wave packet

\[ d = -\alpha f, \quad \alpha \approx \frac{2e r_{at}^2}{E_1 - E_0} \]

\[
f_A = m v \nabla v \approx m \frac{p_0}{m} a^{-1} \frac{p_0}{m} \hat{k}_{rot} \approx \frac{\hbar^2}{ma^3} \hat{k}_{rot}
\]

\[
d_A = -\frac{2e \hbar^2 r_{at}^2}{ma^3(E_1 - E_0)} \hat{k}_{rot} \]

\[ d_{A,B} \approx \pm 10^{-22} \text{ CGSE units} \]
This is certainly true for isolated roton which has zero group velocity in infinite liquid. Near the walls or in presence of normal fluid acceleration in respect of superfluid component the roton wave packet is deformed and the roton acquires dipole moment.
Temperature-Voltage Amplitudes Ratio

\[ f \propto m \frac{\partial(v_s - v_n)}{\partial t} \]

\[ \langle d \rangle = -\alpha f \quad \text{average dipole moment per atom} \]

\[ P = n \langle d \rangle \]

\[ \nabla U = -4\pi P \]

\[ \frac{\partial(v_s - v_n)}{\partial t} = \frac{S}{\rho_n} \nabla T \]

\[ \frac{\Delta T}{\Delta U} = \frac{\rho_n}{4\pi \alpha \rho S} \approx \frac{p_0^2}{12\pi \alpha \rho (\Delta + 3T/2)} = 0.55 \times 10^4 \frac{\text{Kelvin}}{\text{Volt}} \]
Part 2

ROTON
Resonance spectroscopy at roton frequency.
Experimental technique

Basic diagram of measurement in external \textit{dc} electric field.
1 – dielectric disk resonator; 2 – dielectric antennas, 3 – metallic electrode, 4 – part of the metallic body of the cell with HeII, 5 – thermal guns, 6 – d.c. voltage source.

Rybalko, Rudavskii, Rubets et al 2009-2010
Electromagnetic energy distribution in dielectric resonator
Resonance curves of whispering gallery modes of a leucosapphire resonator: a – vacuum, mode $m = 128$, $T = 1.4$ K; b – HeII, mode $m = 128$, $T = 1.4$ K
Resonance absorption. Induced photon radiation.

Thermal gun power

\[ W = 0 \]

\[ W = 7 \text{ watt/cm}^2 \]
FIG. 3. Temperature dependence of the resonance absorption frequency (right-hand axis) and recalculated roton gap value (left-hand axis): •: this experiment and ○: neutron experiment (Ref. 14).
Conservation laws

\[ P_{\text{liquid}} = mn\Omega V = \hbar k_{\text{photon}} - p_{\text{roton}} \approx -p_0 \]

\[ E_{\text{liquid}} = \hbar ck_{\text{photon}} - \Delta = mn\Omega \frac{V^2}{2} = \frac{P_{\text{liquid}}^2}{2mn\Omega} \approx \frac{p_0^2}{2mn\Omega} \approx \frac{\Delta}{n\Omega} \ll \Delta \]
Resonance absorption splitting in electric field

\[ E_{dc} = 4 \text{ kV/cm. } T = 1.8 \text{ K, whispering gallery mode} \quad m=128 \]
Frequency shift as a function of electric field

\[ d_{\text{rotot}} = \frac{hf}{E} \approx 10^{-22} \quad \text{CGSE units} \]
Conclusion

- The second sound waves as well other types of liquid He-II thermal flows characterized by the relative nonstationary motion of normal and superfluid component are accompanied by the dynamic polarization of the liquid.
- The theoretical estimation of ratio of the amplitudes of temperature and electric polarization is in reasonable correspondence with the observations.
- The experimentally registered narrow line of resonance absorption at the roton frequency is split by a constant electric field into two lines as if roton possesses a dipole moment.
- An estimation of the upper limit instantaneous dipole moment of atom involved in atomic motion surrounding single roton is in reasonable agreement with measured “roton” dipole moment.
- The problem deserves further experimental studies: measurements at low temperatures, inelastic neutron scattering in const electric field….., that hopefully stimulates theoretical investigations of microscopic mechanism of polarization phenomena in superfluid $^4$He.