Eliashberg theory of spin-fluctuation pairing in cuprate superconductors

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Dear Sima and Seva:

HAPPY BIRTHDAY!
High Tc cuprates
Parent compounds are antiferromagnetically ordered
Overdoped, but still superconducting materials, are Fermi liquids
Superconducting state has d-wave symmetry
Overdoped compounds are metals and Fermi liquids

\[ \text{Tl}_2\text{Ba}_2\text{CuO}_{6+\delta} \]

Photoemission

Oscillations in resistance/magnetization

Areas are consistent with Luttinger count for electrons in a Fermi liquid \((1+x)\)
The superconducting gap has d-wave symmetry

$$\Delta (\theta) = \Delta_0 \cos 2 \theta$$
From the very early days of high-Tc

What if we replace phonons by antiferromagnetic spin fluctuations, and perform BCS-type calculations?

Analog of BCS

Scalapino et al, Monthoux, Balatsky & Pines, Carbotte…
Magnetic interaction is repulsive

\[ \Delta (k) = -\int d q \frac{\Delta (q)}{\sqrt{\Delta^2 (q) + E^2 (q)}} \chi_{\text{spin}} (k - q) \]

\( \chi_{\text{spin}} (k - q) \) is positive (paramagnetic)

No s-wave solution
Magnetic interaction is repulsive

\[
\Delta (k) = - \int \! \! \! \int \! d^2q \frac{\Delta (q)}{\sqrt{\Delta^2 (q) + E^2 (q)}} \chi_{\text{spin}} (k - q)
\]

Assume that \( \chi_{\text{spin}} (k - q) \) is peaked at \( (\pi, \pi) \).
Magnetic interaction is repulsive

\[ \Delta(k) = - \int \frac{\Delta(q)}{\sqrt{\Delta^2(q) + E^2(q)}} \chi_{\text{spin}}(k - q) \]

Eqn. for a sc gap

Assume that \( \chi_{\text{spin}}(k - q) \) is peaked at \((\pi, \pi)\)

\[ \Delta(x^2 - y^2) \text{ gap} \]
Objections:

- Cuprates near optimal doping (where $T_c$ is the largest) are NOT weakly coupled Fermi liquids
"Non-Fermi liquid" physics
(most likely, Fermi liquid, but with small upper edge)

In a Fermi liquid
\[ \rho(T) \propto T^2 \]

\[ \Sigma''(\omega) \propto \omega \]

\[ \Sigma''/ \propto \omega^2 + (\pi T)^2 \]
Objections:

• Cuprates near optimal doping (where Tc is the largest) are NOT weakly coupled Fermi liquids

• There is pseudogap behavior over a wide range of temperatures
Pseudogap

STM

Bi$_2$Sr$_2$CaCu$_2$O$_8$

$(T_c = 82 \text{ K})$

Ch. Renner et al.
PRL 80, 149 (1998)

ARPES

$(\pi,0)-($\pi,\pi$)$

H. Ding et al.
Nature 382, 51 (1996)

IR: $1/\tau(\omega)$

$1/\tau(\omega)$, cm$^{-1}$

A. Puchkov et al.
PRL 77, 3212 (1996)

Raman

$1/\tau(\omega)$, cm$^{-1}$

G. Blumberg et al.
Science 278, 1427 (1997)
Objections:

• Cuprates near optimal doping (where $T_c$ is the largest) are NOT weakly coupled Fermi liquids

• There is pseudogap behavior over a wide range of temperatures

• Parent compounds of cuprates are not just antiferromagnets, they are also Mott insulators.
Mott insulators

Resistivity

parent compounds (magnetic)

Ando et al
Mott insulators

Optical conductivity

The same Mott gap of 1.7 eV in hole-doped and el-doped materials

Nd$_{2-x}$Ce$_x$CuO$_4$ $E//ab$

- $x=0.15$
- $10K$
- $290K$

- $x=0.125$
- $10K$
- $290K$

- $x=0.10$
- $10K$
- $290K$

- $x=0.05$
- $10K$
- $440K$

- $x=0.0$
- $10K$
- $290K$

Onose et al, Zimmers et al
What is more important, Mott physics or antiferromagnetism? (actually quite relevant to the issue of pairing mechanism)

- Doped Mott insulator is not a Fermi liquid (fermions are almost localized) – new pairing theory is required

  - Doped Heisenberg antiferromagnet is a Fermi Liquid with a small, pocketed Fermi surface – one can study pairing using conventional theories

+ Small amount of holes
What is more important, Mott physics or antiferromagnetism? (actually quite relevant to the issue of pairing mechanism)

- Doped Mott insulator is not a Fermi liquid (fermions are almost localized) – new pairing theory is required

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The crossover from small to large Fermi surface:

Increasing SDW order

A lot of fluctuations around these states, but Fermi liquid survives at $T=0$
Evidence for small Fermi pockets

Suchitra E. Sebastian, N. Harrison, M. M. Altarawneh, Ruixing Liang, D. A. Bonn, W. N. Hardy, and G. G. Lonzarich


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**FIG. 2:** Magnetic quantum oscillations measured in YBa$_2$Cu$_3$O$_{6+x}$ with $x \approx 0.56$ (after background polynomial subtraction). This restricted interval in $B = |B|$ furnishes a dynamic range of $\sim 50$ dB between $T = 1$ and 18 K. The actual $T$ values are provided in Fig. 3.

Original observation:
N. Doiron-Leyraud, C. Proust, D. LeBoeuf, J. Levallois, J.-B. Bonnemaison, R. Liang, D. A. Bonn, W. N. Hardy, and L. Taillefer,

Evidence for small Fermi pockets

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L. Taillefer, cond-mat. 0901.2313
Evidence for antiferromagnetism

- Incommensurate antiferromagnetism
- d-wave superconductor
Evidence for antiferromagnetism
Let’s take antiferromagnetism and change in Fermi surface as the two ingredients of cuprate phase diagram.
Question: can we get high Tc superconductivity?

Onset of $d$-wave superconductivity hides the critical point $x = x_m$. 

- Small Fermi pockets
- Quantum-critical behavior
- Large Fermi surface
- Spin density wave (SDW)
Pairing mediated by strong antiferromagnetic spin fluctuations
An exchange of antiferromagnetic spin fluctuations yields d-wave pairing.

Weak coupling: just replace phonons by spin fluctuations.

\[
T_c \sim \omega_{sf} e^{-(1+\lambda)/\lambda}, \quad \omega_{sf} \sim 20 - 30 \text{ meV (200 - 300 K)} \quad \lambda \leq 1 \Rightarrow T_c \sim 100 \text{K}
\]

It is possible to explain \( T_c \sim 100 \text{ K} \) within weak/moderate coupling theory.

Weak coupling BCS is inconsistent with Fermi liquid. Need to go beyond BCS.
Eliashberg theory of spin fluctuation mediated d-wave superconductivity

Search for “Eliashberg” yielded about 2500 papers only in PRB
Model: fermions coupled to their collective bosonic fluctuations in the spin channel (spin-fermion model)

Ingredients: low-energy fermions \( v_F \), spin excitations \( \xi^{-1} \) and spin-fermion interaction \( g \)

Parameters: one overall energy scale \( g \)
one dimensionless coupling \( \lambda \propto g/v_F \xi^{-1} \)

(mass renormalization \( m^*/m = 1 + \lambda \))

Strong coupling: when \( \xi \to \infty \), \( \lambda \propto \xi \to \infty \)

The scale \( g \) is still assumed to be smaller than the fermionic bandwidth
Why Eliashberg theory?

In bare theory, there is no difference between velocity of fermions and velocity of bosons – both are Fermi velocities, hence no Migdal theorem.

Once you dress up fermions AND bosons by self-energies, bosons become Landau overdamped and hence slow compared to fermions

\[ \Sigma (k_F, \omega) = \lambda \omega, \quad \Sigma (k, \omega = 0) \sim v_F (k - k_F), \]

\[ \partial \Sigma (k, \omega) / \partial \omega = \lambda > 1, \quad \partial \Sigma (k, \omega) / \partial (v_F k) = O(1), \]

Vertex corrections: \[ \Delta g / g = O(1) \]

One needs large \( N \) to put theory under control \([O(1) \rightarrow O(1/N)]\)
• compute normal state fermionic and bosonic propagators (with self-energies included)

• compare with experiments, extract $g$ and $\lambda$

• use the renormalized propagators and Eliashberg theory for the pairing problem, see what $T_c$ and feedbacks from the pairing on electrons we get

At this stage, no free parameters.
Consider first doping region with no Fermi surface reconstruction.

Collective spin excitations can decay into particles and holes and become Landau overdamped.

Fermions acquire a finite damping due to interaction with Landau overdamped collective excitations.
Spin fluctuations in the normal state form a gapless continuum.

\[ \text{Im } \chi (\pi, \omega) \sim \text{Im } \frac{1}{1 - i \omega/\omega_{\text{sf}}} \]

where \( \omega_{\text{sf}} \propto g/\lambda^2 \left( = \frac{9}{64 \pi} \frac{g}{\lambda^2} \right) \)

In units of meV, \( g \sim 1.7 \text{ eV} \) and \( \omega_{\text{sf}} \propto 20 - 30 \text{ meV} \Rightarrow \lambda \sim 1.5 - 2 \)
Fermionic self-energy

$\Sigma(\omega)/\lambda\omega_{sf}$

MFL-like behavior

$\omega_{sf}$

is the upper boundary of the Fermi liquid behavior

Nodal

T=100K
FWHM=54meV

A. Kaminski et al.
PRL 84, 1788 (2000)

Anti-nodal

$\lambda = 2$

Bi2212 OP89K (100K)

Data: Kaminski et al
Conductivity in YBCO$_7$
Quasiparticle dispersion from photoemission

Theory

Experiment
Fermi liquid

Quantum-critical behavior

$\omega_{\text{sf}} \propto \xi^{-2}$

Spin density wave (SDW)

Large Fermi surface

Fermi liquid
The pairing problem

Two candidate scales

\[ \omega_{sf} \sim \frac{g}{\lambda^2} \]

\[ \Sigma(k, \omega) > \omega \]

\[ \Sigma(k, \omega) < \omega \]

energy

- upper boundary of a Fermi liquid
- upper boundary of the strong coupling behavior

\[ T_c \sim \omega_{sf} e^{-(1+\lambda)/\lambda} \sim g \xi^{-2} \]

vanishes at \( \xi = \infty \)

\[ T_c \sim g, \text{ remains finite at } \xi = \infty \]

(if pairing is within a FL)  (if pairing involves fermions outside of a FL)
Pairing in non-Fermi liquid regime is a new phenomenon

This is strong coupling limit of Eliashberg theory

Equation for the pairing vertex $\Phi(\Omega)$ has non-BCS form

\[
\Phi_k(\Omega) = -\frac{\pi}{4} T \sum_{\omega} \frac{\Phi_{k+\pi}(\omega)}{|\omega|^{1/2} |\Omega - \omega|^{1/2}} \frac{1}{1 + (|\omega|/g)^{1/2}},
\]

\[
\Sigma(\omega) \propto \omega^{1/2} \quad \chi_L(\omega) \propto \omega^{-1/2} \quad 1 + \omega / \Sigma(\omega)
\]

This is NOT BCS – summing up logarithms gives no pairing at all

\[
\Phi(\Omega) = \Phi_0 \left( \frac{g}{\Omega} \right)^{1/4} \text{ at } T = 0
\]
Full solution (beyond logarithmic approximation): the pairing instability exists at $T_{\text{ins}} \sim g$

Fermi liquid pairing only, $T_{\text{ins}} \sim \omega_{sf}$

(same as the distance from a magnetic instability)

$T_{\text{ins}} = 0.025 g$

at $\lambda = \infty$ (200 K)

Abanov, A.C, Finkelstein
This problem is actually quite generic.

\[
\Phi (\Omega) = \bar{\lambda} \pi T \sum_\omega \frac{\Phi(\omega)}{|\omega|^{1-\gamma} |\Omega - \omega|^\gamma} \frac{1}{1 + (|\omega|/g)^\gamma}, \quad \bar{\lambda} = \frac{1 - \gamma}{2}
\]

- **\( \gamma = 1/2 \)**
  - Antiferromagnetic QCP
  - Abanov, A.C., Finkelstein, hot spots

- **\( \gamma = 1/3 \)**
  - Ferromagnetic QCP
  - Haslinger et al, Millis et al, Bedel et al...
  - \( \Omega^{2/3} \) problem: gauge field, nematic...

- **\( \gamma = 1/4 \)**
  - 2\( k_F \) QCP
  - Krotkov et al, electron-doped

- **\( \gamma = 2 \)**
  - Pairing by near-gapless phonons
  - Allen, Dynes, Carbotte, Marsiglio, Scalapino, Combescot, Maksimov, Bulaevskii, Rainer, Dolgov, Golubov, ...

- **\( \gamma = +0 \) (log \( \omega \))**
  - 3D QCP, Color superconductivity
  - Son, Schmalian, A.C....

- **\( \gamma = 1 \)**
  - Z=1 pairing problem

- **\( \gamma = +0 \rightarrow \gamma = 1 \)**
  - Pairing in the presence of SDW
  - Moon, Sachdev

- **\( \gamma \approx 0.7 \)**
  - Fermions with Dirac cone dispersion
  - Metzner et al
Quantum-critical behavior

Spin density wave (SDW)

Large Fermi surface

Wave conductor

$x_m$
Pairing in the presence of Fermi surface reconstruction

Increasing SDW order

Pairing by spin waves, relevant scale is $J \sim 100$ meV

$T_c$ decreases due to the reduction in spin-fermion vertex

Numbers match – their $E_F$ for POCKETS is 10 times smaller than $g$
Quantum-critical behavior

Small Fermi pockets with pairing fluctuations

Large Fermi surface

d-wave superconductor

Spin density wave (SDW)
Story is far from completion:

- Linear in resistivity is NOT explained

Theory: resistivity in the critical regime is linear in $T$ only above some $T_0$
• The nature of the pseudogap phase is not fully understood

It is natural to associate this phase with SDW precursors (fluctuating pockets), but

• There surely are superconducting fluctuations above $T_c$

• There are experimental evidences for potential discrete symmetry breaking in the pseudogap phase
Neutron scattering: breaking of rotational symmetry

Could be pre-emptive ordering of Ising degree of freedom associated with incommensurate magnetic order \([(Q, \pi) \text{ or } (\pi, Q)]\), but this remains to be seen.
Conclusions

Recent experiments show that cuprates are more “conventional” than previously thought

- Coherent fermionic excitations are present at all dopings
- Long-range magnetic order extends up to optimal doping

This gives weight to the scenario that the pairing in the cuprates is mediated by near-critical spin fluctuations, and from this perspective is not that different from the pairing in heavy-fermion and organic superconductors.
Quantum-critical behavior

Small Fermi pockets with pairing fluctuations

Large Fermi surface

d-wave superconductor

Spin density wave (SDW)
Collaborators

- Artem Abanov (LANL/Texas)
- Sasha Finkelstein (Weizman/Texas A&M)
- Rob Haslinger (LANL)
- Dirk Morr (Illinois-Chicago)
- Joerg Schmalian (Iowa)
- Mike Norman (Argonne)
- Pavel Krotkov (Maryland)
- Ilya Eremin (Dresden)
- Karl Bennemann (Berlin)
- Oleg Tchernyshov (Baltimore)
- Boldizsar Janko (Notre Dame)
- David Pines (UC Davis)
- Philippe Monthoux (Edinburg)
- Matthias Eschrig (Karlsruhe)
- Tigran Sedrakyan (Maryland)
- A. Millis (Columbia)
- E. Abrahams (UCLA)
- S. Maiti (Wisconsin)
- D. Dhokarh (Wisconsin)
Whatever your wish...
I hope that is the best for you

Happy Birthday!